INTERNATIONAL MATHEMATICS TOURNAMENT OF TOWNS

JUNIOR PAPER: YEARS 8,9,10

Tournament 40, Northern Autumn 2018 (O Level)

Note: Each contestant is credited with the largest sum of points obtained for three problems.

1. Let ABC be a right-angled triangle with $\angle B = 90°$. A circle through B and the midpoint K of hypotenuse AC intersects the sides AB and BC at points M and N respectively. Suppose that $AC = 2MN$. Prove that M and N are the midpoints of the sides AB and BC respectively. (4 points)

2. Determine all positive integers n such that the numbers $1, 2, \ldots, 2n$ can be divided into pairs so that the product of sums of the numbers in each pair is a perfect square. (4 points)

3. A grid rectangle of the size 7×14 is divided along the grid lines into the squares of the size 2×2 consisting of 4 squares and corners consisting of 3 squares. Is it possible that the number of squares of the size 2×2 is

   (a) equal to the number of corners? (1 point)

   (b) greater than the number of corners? (3 points)

4. Nastya has 5 coins, which look identical, three of which are real and of the same weight, and the other two are fake. Of the fakes, one weighs more than a real coin, and the other weighs less than a real coin by the same amount. Nastya can ask an expert to perform three weighings of her choice on a simple balance. Then the expert reports the results to Nastya. Note that the results of all weighings are reported to Nastya after the third weighing. Could Nastya choose the weighings so that she would be able to determine both fake coins and state which of them is heavier for sure? A simple balance shows which of two sides is heavier/higher or if they are balanced. (5 points)

5. A nine-digit integer is called *beautiful* if all of its digits are different. Prove that there exist at least 1000 beautiful numbers, each of which is divisible by 37. (5 points)